STRUCTURE FUNCTIONS AND LOW X PHYSICS

by

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Abstract

In the framework of conventional Regge theory we present a common description of total photon-proton cross-section and proton structure functions in the region $0 \leq Q^2 \leq 5$ GeV^2. Using it as an initial condition in the perturbative QCD evolution equation, we describe structure functions at any Q^2 and x - including the small x region explored at HERA.

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Perturbative QCD predicts the Q^2-dependence of structure functions of hadrons and nuclei when the initial conditions (i.e. the momentum distribution functions of quarks and gluons at $Q^2 = Q_0^2$) are given. The value of Q_0^2 has to be large enough for the perturbative expansion (which is known in two-loops) to be valid. The determination of the initial condition is a problem of soft (non-perturbative) physics. This problem is especially important in the region of very small x which is now experimentally accessible at HERA. Indeed, the study of the region $x << 1$ (Regge limit) and large Q^2, should help to clarify the interplay between perturbative and non-perturbative physics.

In this letter, we use the present knowledge of soft diffractive processes to formulate the initial condition for perturbative QCD evolution. Our aim is to obtain a simple, theoretically motivated description of both the total γp cross-section and the structure function in a region of low and moderate values of Q^2 ($0 \leq Q^2 \leq 5$ GeV^2), covering the full x-region. Using this initial condition in the perturbative QCD evolution equation we obtain the structure functions at larger values of Q^2. Our results are in agreement with experiment in the whole Q^2 and x region.

Small x behaviour of the structure functions In Regge theory the high-energy behaviour of hadron-hadron and photon-hadron total cross-sections is determined by the Pomeron intercept $\alpha_P = 1 + \Delta$, and is given by $\sigma^{tot}_{\gamma(h)p}(\nu) \sim \nu^{\Delta}$. This behaviour is also valid for a virtual photon for $x = Q^2/2m\nu << 1$, leading to the well known behaviour, $F_2(x, Q^2) \sim x^{-\Delta}$, of the structure functions at fixed Q^2 and $x \to 0$.

In many analysis of the structure functions either $\Delta = 0$ or $\Delta = 0.5$ have been used[1]. The first value corresponds to a constant cross-section and sometimes it is wrongly associated with the intercept of the soft Pomeron. The second value corresponds to an estimate of the hard (Lipatov) Pomeron intercept. Both choices are in disagreement with our knowledge of the Pomeron intercept inferred from a study of the energy dependence of hadronic cross-sections. This behaviour is universal and given by ν^{Δ_0} with $\Delta_0 \approx 0.08$[2].

However, due to the existence of absorptive corrections (rescattering), this is not the true Pomeron intercept (sometimes referred to as "bare" Pomeron), but rather an effective one. The "bare" Pomeron intercept is in fact substantially larger than $1 + \Delta_0$. In the generalized eikonal model (with diffractive intermediate states taken into account) one gets $\Delta_1 = 0.12 \div 0.14$[3,4]. In a theory containing a more complete set of absorptive graphs, one obtains an even larger value $\Delta_2 = 0.2 \div 0.25$[5], leading to the "bare" Pomeron intercept $1 + \Delta_2$. These considerations are very important if one attempts to cover a broad region of Q^2 which includes also the real photon ($Q^2 = 0$). Indeed the relative contribution of the most important absorptive corrections decreases quite rapidly when Q^2 increases (like Q^{-2} for the eikonal ones), so that as $Q^2 \to \infty$ we see a Pomeron intercept which is close to the "bare" one $1 + \Delta_2$ - much larger than the effective intercept $1 + \Delta_0$ seen at $Q^2 = 0$. These considerations have prompted us to use a low x behavior of the structure function of the form

$$F_2(x, Q^2) \sim x^{-\Delta(Q^2)} \quad ; \quad \Delta(Q^2) = \Delta_0 \left(1 + \frac{2Q^2}{Q^2 + d}\right) \tag{1}$$

where Δ_0 and d are free parameters. In the fit described below one obtains $\Delta(Q^2 = 0) = \Delta_0 \sim 0.08$, so that $\Delta(Q^2 \to \infty) \sim 0.24$, corresponding to the "bare" Pomeron intercept $1 + \Delta_2$, as discussed above.

Structure functions at low Q^2 In order to obtain the initial condition for perturbative QCD evolution we have to know the structure functions at $Q^2 = Q_0^2$ and all values of x. Furthermore it is necessary to know separately the contributions of valence and sea quarks and gluons. According to the large-N expansion, we assume that the Pomeron determines the small-x behaviour of sea quarks and gluons (eq. (1)), while secondary reggeons (ρ, ω, f, A_2), with intercept $\alpha_R \sim 0.4 \div 0.6$, determine that of valence quarks.

In order to include real photons ($Q^2 = 0$), we have to use the relation

$$\sigma_{\gamma p}^{tot}(\nu) = \left[\frac{4\pi^2\alpha_{EM}}{Q^2}F_2(x,Q^2)\right]_{Q^2=0} \quad , \tag{2}$$

which implies that $F_2(x,Q^2) \sim Q^2$ at $Q^2 \to 0$. With these considerations in mind we propose the following parametrization of the structure functions in the region of small and moderate $Q^2 (0 \leq Q^2 \leq 5 \text{ GeV}^2)$:

$$F_2(x,Q^2) = A\ x^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4}\left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)}$$
$$+B\ x^{1-\alpha_R}(1-x)^{n(Q^2)}\left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R} \quad . \tag{3}$$

The first term in (3) corresponds to the Pomeron contribution with an $x \to 0$ behaviour given by eq. (1). The second term corresponds to the secondary reggeon contribution with an $x \to 0$ behaviour determined by the secondary reggeon intercept α_R. The behaviour at $x \to 1$ is given by the second factor, with $n(Q^2)$ parametrized as

$$n(Q^2) = \frac{3}{2}\left(1+\frac{Q^2}{Q^2+c}\right) \quad , \tag{4}$$

in such a way that, at $Q^2 = 0$, we have the same power $(1-x)^{1.5}$ as in the Dual parton model[6] (also controlled by Regge intercepts : $n(0) = \alpha_R(0) - 2\alpha_N(0) \approx \frac{3}{2}$), and at $Q^2 \to \infty$ we have $(1-x)^3$ as given by dimensional counting rules. The last factor in the two terms of eq. (3) is required by eq. (2) in order to connect with real photons. (A similar factor has been previously introduced in the same physical context ; see [7], [8] and references therein). Using eq. (2) one has :

$$\sigma_{\gamma p}^{tot}(\nu) = 4\pi^2\alpha_{EM}\left(A\ a^{-1-\Delta_0}(2m\nu)^{\Delta_0} + B\ b^{-\alpha_R}(2m\nu)^{\alpha_R-1}\right) \quad . \tag{5}$$

In this way we have a parametrization of both structure functions and γp total cross-section with 8 free parameters. Four of these parameters appear in (5) so that the structure function of a proton contains only 4 extra parameters. Out of these only three are free

since we fix the parameter B using the normalization condition for valence quarks. In order to do so we have first to separate in the second term of (3) the contribution of the u and d valence quarks. This is done by replacing the term $B(1-x)^{n(Q^2)}$ in eq. (3) as follows :

$$B(1-x)^{n(Q^2)} \to B_u(1-x)^{n(Q^2)} + B_d(1-x)^{n(Q^2)+1} \quad . \tag{6}$$

The condition of having in the proton two valence u-quarks (charge $2/3$) and one valence d-quark (charge $-1/3$), determines the values of B_u and B_d (see below).

The seven remaining parameters were determined from a joint fit of the $\sigma_{\gamma p}^{tot}$ data and the new NMC data[9] on the proton structure function $F_2(x,Q^2)$ in the region 1 GeV$^2 \leq Q^2 \leq 5$ GeV^2. The description of the data is quite good (the χ^2 is practically the same as the one of the parametrization used in ref. [9] with 15 free parameters, and much better than the one of the parametrization of ref. [8] with 20 free parameters). Our results are shown in Figs. 1 and 2 and the values of the parameters (which are strongly correlated) are given in the caption of Fig. 2. A comparison of our predictions for $F_2(x,Q^2)$ with the SLAC data[10] (not included in the fit) is shown in Fig. 3. The agreement is quite good.

As mentioned above our parametrization gives the separate contributions of valence and sea quarks. Thus, we can predict the structure function $F_2(x,Q^2)$ of a neutron (deuteron). In Fig. 4 we compare our predictions for the structure function of a deuteron with experimental data[9,10]. The agreement is very good. Likewise, structure functions for $\nu(\bar{\nu})N$-scattering are also well described[11].

In order to determine the distribution of gluons in a nucleon we assume that the only difference between distributions of sea-quarks and gluons is in the $x \to 1$ behaviour. Using the arguments of ref. [12] we write it in the form

$$x\ g(x,Q^2) = Gx\bar{q}(x,Q^2)/(1-x) \quad , \tag{7}$$

where $x\bar{q}(x,Q^2)$ is proportional to the first term of eq. (3). The constant G is determined from the energy-momentum conservation sum rule. Comparison of our predictions with the available data is shown in Fig. 5.

Structure functions at large Q^2 We are now ready to introduce the QCD-evolution in our partonic distributions and thus to determine structure functions at larger values of Q^2. We use the evolution equation in two loops in the $\overline{MS}$ scheme with $\Lambda = 200$ MeV.

For the initial point Q_0^2 we can use any value in the region 1 GeV$^2 \leq Q^2 \leq 5$ GeV^2. However, since our initial condition describes the data in the whole region $Q^2 \leq 5$ GeV^2, we know not only the function $F_2(x,Q_0^2)$ but also its derivative $\left.\frac{dF_2(x,Q^2)}{d\ \ell nQ^2}\right|_{Q^2=Q_0^2}$, which in general will not coincide with that obtained from the QCD evolution equation. We checked, however, that for $Q_0^2 \approx 2$ GeV^2, these two derivatives are very close to each other in a broad region of x. So we have chosen as an initial condition $Q_0^2 = 2$ GeV^2 in order to obtain a smooth behaviour in Q^2. Actually, it is possible to get an exact equality of derivatives if, for $Q^2 \geq Q_0^2$, we add to the QCD-evolution a term decreasing as a power of Q^2 (higher twist) :

$$F_2(x,Q^2) = F_2^{pert}(x,Q^2)\left(1+\frac{f(x)}{Q^2}\right) \qquad Q^2 \geq Q_0^2 \quad . \tag{8}$$

The function $f(x)$ is determined by requiring the equality of the Q^2-derivatives of our initial condition (3) and of eq. (8) at the point Q_0^2. The results obtained using (8) are shown in Figs. 2-4 as full curves. The agreement with the data is quite good.

Although our fit of the NMC data has been restricted to the region 1 GeV$^2 \leq Q^2 \leq$ 5 GeV^2, it is interesting to mention that a good fit can also be obtained up to $Q^2 \sim$ 10 GeV^2. This produces only small changes in our parameters and allows to start the perturbative QCD evolution at larger values of Q^2. Details of such a parametrization will be given elsewhere.

The low x HERA region Our predictions for the small x-region measured at HERA[13,14] are shown in Figs. 6, 7. It follows from these figures that the initial condition, eq. (3), based on our knowledge of the Pomeron properties inferred from soft processes, together with conventional QCD evolution, can explain the increase at very small x observed at HERA[13,14].

This is obtained with a Pomeron intercept $1 + \Delta(Q^2)$ with $\Delta(Q^2 \to \infty) = 0.2 \div 0.25$. This is about half way between the values $\Delta = 0$ and $\Delta = 0.5$ used in the literature at large Q^2. Since this value of the Pomeron intercept is consistent with the one obtained in the study of high energy hadronic interactions, we consider that our results are in favor of the idea[6,15−17] that there is only one Pomeron (rather than a soft and a hard ones). For discussion on this point see for example ref. [18].

In our approach the low-x increase of the structure functions observed at HERA is largely due to the initial condition. This approach is orthogonal to the one in ref. [19] where the low-x rise at HERA is entirely due to the perturbative QCD evolution (started at $Q^2 = Q_0^2 \sim 0.3$ GeV^2). An intermediate situation occurs in the approaches of ref. [20]. Forthcoming data from the E665 collaboration at Fermilab in the unexplored region of low x and low Q^2 should provide a clear cut distinction among these approaches.

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Figures Captions

Fig. 1 The data for total γp cross-section versus $\sqrt{s}$, including the high-energy HERA ones, are compared with the result of a joint fit of $\sigma_{\gamma p}$ and NMC proton structure functions described in the main text.

Fig. 2 The proton structure function $F_2(x, Q^2)$ obtained by the NMC collaboration[9] at 90 GeV (full circles) and 280 GeV (open circles). The dashed line is obtained from eqs. (3) and (6) with the following values of the parameters (all dimensional parameters are in GeV^2) : $A = 0.1502$, $a = 0.2631$, $\Delta_0 = 0.07684$, $d = 1.1170$, $b = 0.6452$, $\alpha_R = 0.4150$, $c = 3.5489$. (The values of the parameters B_u and B_d in (6), determined from the normalization condition for valence quarks (at $Q^2 = 2\ \mathrm{GeV}^2$), are : $B_u = 1.2064$ and $B_d = 0.1798$. The full line is obtained using QCD evolution (see eq. (8)).

Fig. 3 Same as in Fig. 2 except that the data are now from SLAC[10].

Fig. 4 Same as in Figs. 2 and 3 for the deuteron structure functions. Here $B_u = 0.7540$ and $B_d = 0.4495$.

Fig. 5 The gluon structure function at $Q^2 = 9\ \mathrm{GeV}^2$. The theoretical curve is obtained from eq. (7).

Fig. 6 The x-dependence of the proton structure function at three different values of Q^2. The lower (upper) curve for $Q^2 = 15\ \mathrm{GeV}^2$ is obtained using eq. (8), with (without) the higher twist term.

Fig. 7 The Q^2-dependence of the proton structure function for different values of x, compared with HERA data.

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